Measurement Error and the Specification of W

Garrett N. Vande Kamp

June 28, 2018

Spatial regression models are becoming increasingly popular across the social sciences as a means of modeling spatial dependencies within data. At the core of these models is **W**, a connectivity matrix between observations. Despite the centrality of **W** in spatial regression models, however, there is a scarcity of techniques for evaluating the validity of a given specification of **W**. I argue that approaching the specification of **W** as a measurement error problem leads to some important insights. I demonstrate that when **W** is misspecified, a predictable form of omitted variable bias occurs. I also construct a theoretically appealing test for the validity of **W** which, while intractable in cross-sectional settings, is easily applied in settings featuring multiple cross-sections like panel data. I demonstrate the validity of this test using simulations with an examination of the models in Williams and Whitten (2015). By demonstrating the utility of the approach, I simultaneously provide scholars a means of testing their modelling assumptions and advance a larger theoretical framework for dependence across data that will be fruitful for future research.

The social sciences are full of theories that argue units of analysis are spatially dependent upon one another, including political science. U.S. states learn from each through policy diffusion. Legislators take into account their peers' votes when making their own choices. Countries make choices based on those of their neighbors and their allies. The list goes on (for a review, see Hays, Kachi, and Franzese 2010).

Spatial regression models are becoming increasingly popular as a means of modeling spatial dependencies within data. At the core of these models is the spatial weights matrix $\mathbf{W}$, which contains the set of spatial relationships between all units in the data. Despite the centrality of $\mathbf{W}$ in spatial regression models, however, there is a scarcity of techniques for testing the validity of a particular choice of $\mathbf{W}$. While a number of methods exist for choosing between a set of different specifications of $\mathbf{W}$, few techniques exist for evaluating the validity of a particular choice of $\mathbf{W}$. Those that do are restricted to a subset of potential $\mathbf{W}$ specifications and are unable to validate the relative distance between observations in the sample. What is more, some scholars downplay the importance of a correct specification of $\mathbf{W}$, arguing that only in cases of severe misspecification of $\mathbf{W}$ will inferences be threatened (LeSage and Pace 2014). For all of the importance of $\mathbf{W}$ in the estimation of spatial models, scholars lack the tools for assessing the validity of $\mathbf{W}$ and are advised not to care.

This paper attempts to both explain the bias resulting from the misspecification of $\mathbf{W}$ and provide a test for the validity of a particular specification of $\mathbf{W}$. This is done by framing the specification of $\mathbf{W}$ as a measurement error problem. Specifically, each of the $w_{ij}, i \neq j$, terms in the matrix is misspecified. While misspecification of $\mathbf{W}$ will invariably lead to attenuation bias like in most cases of measurement error in the independent variable(s), misspecification of $\mathbf{W}$ will also lead to other forms of bias that are not normally a problem for other forms of measurement error.

Recognizing the misspecification of $\mathbf{W}$ as a measurement error problem, however, reveals a theoretically appealing regression-based test of the specification of $\mathbf{W}$, which I call the

K test, using a **W** matrix comprised entirely of ones. Monte Carlo experiments show that the test performs well in a variety of different **W** misspecifications. Unfortunately, the test cannot be used in a single cross-section. It can, however, be used in settings with multiple cross-sections, such as panel data settings by using tools readily available to scholars. I demonstrate this by examining the models in Williams and Whitten (2015), finding evidence of misspecified spatial relationships in one of the models. By demonstrating the utility of the approach, I advance the larger theoretical framework of multiplicative interactions and reveal additional avenues of research for scholars.

1 W and the Challenge of Its Specification

Spatial regression models are designed to address dependencies in space, broadly defined, in a data-generating process. Like their time-series counterparts, these models can address dependencies in the dependent variable using a spatial autoregressive model (SAR), in the independent variable using a spatial lag of **X** (SLX), and in the disturbance term using a spatial error model (SEM). But while the unidimensional nature of time makes time series models relatively easy to specify using a temporal lag, the multidimensional and multidirectional nature of space makes the creation of spatial lags more difficult.

This problem is solved by the **W** matrix. **W** is a n by n matrix with zeroes along the diagonal that, in the purely cross-sectional context, contains all of the n(n-1) relationships in the data in the off-diagonal elements: each element w_{ij} represents the influence unit j has on unit i. More extreme values of w_{ij} imply a stronger spatial relationship; likewise, all elements of the matrix containing zeroes imply that two units do not have a relationship. **W** must be specified in order to run diagnostic tests of spatial autocorrelation, such as Moran's I. **W** must also be specified in order to estimate spatial regression models, whether using an instrumental variable approach or a maximum-likelihood approach; when a variable is premultiplied by **W**, the resulting vector is a spatial lag similar to temporal lags in the time-series context.

There are a range of approaches to developing a specification of **W**, both theoretical and empirical in nature (Neumayer and Plumper 2016, Vega and Elhorst 2015, Corrado and Fingleton 2012). Relatively few techniques, however, exist for evaluating a particular specification, surprising considering the importance of the matrix in spatial regression models. Some techniques have been developed for adjudicating between different possible **W** matrices, including the J test, Bayesian model averaging, and approaches using information criteria. (Leenders 2002, Stakhovych and Bijmolt 2009, LeSage and Fischer 2008). Similar techniques have been developed to combine multiple **W** matrices (Parent and LeSage 2008). But these approaches are limited in that they can only help judge the best **W** among a set of choices. These tests do not give any indication that any of these specifications are valid independent of comparison choices, leading to the possibility of picking the best out of a bad set of choices for the specification of **W** (Harris et al. 2011, Leenders 2002).

Neumayer and Plumper (2016) offer a test to determine if the assumptions of the absolute relevance of spatial relationships in **W** are valid. Within **W**, any w_{ij} that equals zero is an indication that observation j has no direct influence on observation i. This is a large assumption to make, large enough that Neumayer and Plumper argue that it needs to be tested. They offer the following model to be tested in order to determine the validity of the absolute relevance assumptions within **W**:

$$\boldsymbol{y} = \boldsymbol{x}\beta + \boldsymbol{W^1}\boldsymbol{x}\rho_1 + \boldsymbol{W^2}\boldsymbol{x}\rho_2 + \boldsymbol{\varepsilon}$$

in which the off-diagonal elements of $\boldsymbol{W^2}$ equal one if the corresponding elements of $\boldsymbol{W^1}$ equal zero and zero otherwise. If $\boldsymbol{W^1}$ has correctly determined the absolute relevance of spatial relationships, then ρ_2 should equal zero. A rejection of this null indicates misspecification of $\boldsymbol{W^1}$ by omitting spatial relationships where they exist.

While the logic of this test is sound, it is limited to only testing the validity of the absolute relevance determinations within **W**. It does not test the validity of the relative relevance assumptions made within **W**, or the relative distance between observations in the data. Neumayer and Plumper (2016) highlight that the elements of **W** are sensitive to most transformations because they change the relative relevance of spatial relationships. If observation i and j have a spatial relationship of 0.25 and observation i and k have a relationship of 0.5, then observation k has twice the influence on observation i compared to observation j. But if a constant of 0.5 were added to both relationships, then observation k would only have half as much influence on observation i as observation j. Indeed, any transformation to **W** outside the multiplication of a constant will result in a change of the relative relevance of the observations. In particular, Neumayer and Plumper (2016) note that, in contrast to other independent variables in a model, "a constant added to or subtracted from [**W**] cannot be absorbed in the intercept".

This makes their test only a partial test of the validity of **W**, as it cannot validate the relative relevance determinations within **W**. Furthermore, the test is inapplicable to those specifications of **W** that do not include any instances in which observation j has absolutely no influence on observation i, such as those based on inverse distance. Thus while the use of this test is clear, more work still needs to be done in assessing the validity of a particular specification of **W**.

Not all scholars share the concern about potential misspecifications of **W**. LeSage and Pace (2014) argue that because competing specifications of **W** are often highly correlated, the resulting substantive inferences from models with different **W** will be similar and thus misspecification of **W** is not usually a problem. Furthermore, they argue that changes in parameter estimates with competing specifications of **W** should not be concerning: "[e]ven if misspecification of the weight matrix leads to incorrect estimates of β and ρ in spatial regression models, the impact of this on inferences regarding the partial derivative response of the dependent variable to changes in the explanatory variables is unclear." They highlight that flexible model specifications can sometimes even make up for potential bias with incorrectly specified **W** matrices, though they note that this will not always fix the problem.

Certainly, some light needs to be shed on whether misspecification of **W** will in fact lead to bias in our estimates of partial derivatives and marginal effects. But even if it is true that inferences about our independent variables are not affected by misspecification of **W**, such an approach is problematic for two reasons. First, the argument is inapplicable to SLX models. Because these models are non-recursive, the marginal effect of a spatial lag of the independent variable in a linear model is equal to the estimated parameter. Any bias in the estimation of these parameters will necessary result in uncorrected bias in subsequent inferences.

Second, misspecification of **W** is problematic from a theory-building perspective. **W** represents "an explicit hypothesis about the strength of inter-location connection" between observations (Corrado and Fingleton 2012). A misspecification of **W**, even if it leads to similar substantive inferences for other independent variables, will lead to a misattribution as to what exactly drives the spatial processes at work in our data. The scientific enterprise should not treat spatial dependencies as mere nuisances but social processes of substantial theoretical interest in their own right. Thus, a theoretically-driven approach to the scientific enterprise requires the full investigation of the specification of **W**.

This is particularly true as scholars recognize that space is more than just geography. Geography is highly correlated with a number of other interesting social phenomena including ideology and affluence. If scholars are only concerned with the effect of independent variables in the model, then a **W** specified using geography may very well suffice. But it would surely be a shame to attribute spatial dependencies in the data to geography or any other process not independently validated using a statistical test.

2 The Specification of W as a Measurement Error Problem

I argue that the specification of **W** should be viewed as a measurement error problem. If **W** is correctly specified, both the individual weights w_{ij} as well as the whole matrix **W** have been measured by scholars without error. When **W** is misspecified, however, measurement error is present in the model in a way that will bias both estimates and inferences. Scholars regularly take steps to combat some general types of measurement error in independent variables. Unfortunately, however, these steps are insufficient for dealing with the complex measurement error associated with the misspecification of **W**. By recognizing this measurement error problem, however, one can develop a theoretically-appealing test for the specification of **W**. To begin, I examine measurement error issues in a simple regression context. Consider the following data-generating process:

$$\boldsymbol{y} = \boldsymbol{x}^{*}\beta + \boldsymbol{\varepsilon}$$

in which $\boldsymbol{\varepsilon}$ is a well-behaved error term. Suppose a scholar would like to model the relationship between $\mathbf{x^*}$ and $\mathbf{y}$ in this data-generating process. However, the scholar does not have access to the independent variable $\mathbf{x^*}$. Instead, the scholar has access to $\mathbf{x}$, a function of $\mathbf{x^*}$ according to the data-generating process:

$$\boldsymbol{x} = \boldsymbol{x}^{*} + \boldsymbol{u}^{*}$$

in which $\mathbf{u^*}$ is measurement error with mean c that is uncorrelated to $\mathbf{x^*}$. While this is the most succinct notation, it will be useful for our purposes to decompose $\mathbf{u^*}$ into the part of the measurement error that will change the expectation of x relative to $\mathbf{x^*}$ and the part that will not. This is achieved as follows:

$$\boldsymbol{x} = \boldsymbol{x}^* + (\boldsymbol{u}^* - c) + c = \boldsymbol{x}^* + \boldsymbol{u} + c$$

in which **u** has a mean of zero but is otherwise is similarly distributed to **u** and c is a constant. By substituting the measurement error just described into the original data-generating process, we observe the problems associated with measurement error in the independent variable.

$$\boldsymbol{y} = \beta(\boldsymbol{x} - \boldsymbol{u} - c) + \boldsymbol{\varepsilon}$$

$$\boldsymbol{y} = \boldsymbol{x}\beta + (\boldsymbol{\varepsilon} - \boldsymbol{u}\beta - c\beta)$$

Here, the scholar notes that there are two sources of endogeneity in the model. The first results because $u\beta$ is contained within the error term. **u** and **x** are necessarily correlated given the above data-generating process of **x**. For this reason, the estimate of β will be biased towards zero; this bias is commonly known as attenuation bias. While scholars have developed a number of approaches to circumventing attenuation bias, most of which are detailed in standard regression textbooks. In general, however, the problem of attenuation bias is rarely solved in applied social science literature, and scholars settle for the idea that while coefficients may be biased towards zero, hypothesis tests will not.

The second source of endogeneity results because $c\beta$ is contained within the error term. This results in the expectation of the error term being nonzero, leading to biased estimates. This source of endogeneity, however, is easily solved by applied scholars by including a constant in the regression model. By doing so in the current example, the scholar would estimate the following equation:

$$\boldsymbol{y} = \alpha + \beta(\boldsymbol{x} - \boldsymbol{u} - c) + \boldsymbol{\varepsilon}$$

$$\boldsymbol{y} = (\alpha - c\beta) + \boldsymbol{x}\beta + (\boldsymbol{\varepsilon} - \boldsymbol{u}\beta)$$

In this equation, the non-zero expectation of the error term is transferred to the coefficient of the constant. As a consequence, the estimate of the constant is biased. This is rarely problematic for scholars, however, because the interest is not in the relationship between the dependent variable and the constant, but the relationship between the dependent variable and independent variables of interest. Indeed, the inclusion of the constant remedies a source of bias for β resulting from a non-zero expectation of the error term. Thus while the models scholars generally employ are not immune to measurement error with an expectation of 0, they do solve for part of measurement error with a non-zero expectation by including a constant in their regression model.

2.1 The Problem of Measurement Error Within W

As we have just seen, measurement error in an independent variable can lead to biased estimates and inferences. Part of the problem, however, can be remedied by including a constant within the regression model. Unfortunately, this quick fix will not alleviate any of the problems associated with the misspecification of **W** in a spatial regression model. To demonstrate this, consider a simple SLX model:

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^{*}\beta + \boldsymbol{W}^{*}\boldsymbol{x}^{*}\theta + \boldsymbol{\varepsilon}$$

in which ε is a well-behaved error term. Suppose a scholar would like to model the relationship between $\mathbf{x*}$ and $\mathbf{y}$ in this data-generating process, both the effect of an observation of $x*_i$ on y_i and on y_j. However, the scholar does not have access to the spatial weights matrix $\mathbf{W*}$. Instead, the scholar has access to $\mathbf{W}$, a misspecification of $\mathbf{W*}$ according to the data-generating process:

$$\boldsymbol{W} = \boldsymbol{W}^* + \boldsymbol{U}^*$$

in which **U*** is hollow matrix of measurement error with the non-diagonal elements **u*** having mean c and are uncorrelated with the nondiagonal elements of **W***. As with the previous example, it will be useful for our purposes to decompose **U*** into the part of the measurement error that will change the expectation of the elements of **W** relative to **W*** and the part that will not. This is achieved as follows:

$$\boldsymbol{W} = \boldsymbol{W}^* + (\boldsymbol{U}^* - \boldsymbol{C}) + \boldsymbol{C} = \boldsymbol{W}^* + \boldsymbol{U} + \boldsymbol{C}$$

in which **U** is a hollow matrix with the non-diagonal elements **u** having a mean of zero but is otherwise is similarly distributed to **u*** and **C** is a hollow matrix where all non-diagonal elements are equal to c. By substituting the measurement error associated with **W** into the spatial data-generating process, we observe the unique problems created by measurement error within **W**.

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + (\boldsymbol{W} - \boldsymbol{U} - \boldsymbol{C})\boldsymbol{x}^*\theta + \boldsymbol{\varepsilon}$$

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + \boldsymbol{W}\boldsymbol{x}^*\theta + (\boldsymbol{\varepsilon} - \boldsymbol{U}\boldsymbol{x}^*\theta - \boldsymbol{C}\boldsymbol{x}^*\theta)$$

As with before, there are two sources of endogeneity within the error term that will affect our inferences. The first is the commonly described attenuation bias. Because of the correlation between the elements of **W** and **U**, **Wx*** and **Ux*** will also be correlated. While scholars have yet to consider attenuation bias specifically in the spatial regression context, it is unlikely to be solved because of the problems scholars have noted in solving attenuation bias in the simpler regression context.

The second source of endogeneity is derived from the matrix **C** and its non-diagonal elements c. Unlike in the simple regression model, **C** is post-multiplied by **x*** to create a new variable. Thus any change to the expectation of the error term is dependent upon the expectation of $\mathbf{x^*}$. As in the previous context, any change in the expectation of the error term will be absorbed into the constant.

That aside, however, there will be additional variation in the error term due to the presence of $-\boldsymbol{C}\boldsymbol{x}^*\theta$. This variation cannot be absorbed into the constant. Further, this variation will necessarily be correlated with $\mathbf{x^*}$ since $-\boldsymbol{C}\boldsymbol{x}^*\theta$ is a linear transformation of the total of $\mathbf{x^*}$ within a cross-section minus the influence of x_i^*. Thus, there will be additional bias in $\mathbf{x^*}$ in the model that is unique to the spatial regression context.

The problems associated with this bias are slightly different in other spatial models. In the SAR model, the term $-\boldsymbol{C}\boldsymbol{y}\theta$ will be present in the error term. As $\mathbf{y}$ is a function of all other variables in the model, the term will be correlated with all other independent variables in the model. Thus the bias that only affects $\mathbf{x^*}$ in the SLX context will affect all variables in the SAR context. In the SEM, the term $-\boldsymbol{C}\boldsymbol{\varepsilon}\theta$ will be in the error term. While this does not necessarily affect coefficient estimates in the model, it will reduce efficiency because there will be unmodeled spatial dependence in the error term.

2.2 A Test for the Misspecification of W

The misspecification of $\mathbf{W}$ in spatial regression models is a particularly pernicious form of measurement error because it introduces additional endogeniety that is not present under normal measurement error conditions. This additional endogeneity, however, is also an opportunity for scholars to develop a specification test of $\mathbf{W}$ by attempting to model this new source of endogeneity. For the SLX data-generating process given above, the model is equivalent to:

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + \boldsymbol{W}^*\boldsymbol{x}^*\theta + \boldsymbol{K}\boldsymbol{x}^*0 + \boldsymbol{\varepsilon}$$

in which **K** is a n*n matrix with 0's along the diagonal and 1's populating the non-diagonal elements. Because the new term **Kx*** has a coefficient of 0, it does not influence y and is thus equivalent to the original model. But if a scholar only had **W** instead of **W*** and modelled this relationship, one would get the following result:

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + (\boldsymbol{W} - \boldsymbol{U} - \boldsymbol{C})\boldsymbol{x}^*\boldsymbol{\theta} + \boldsymbol{K}\boldsymbol{x}^*0 + \boldsymbol{\varepsilon}$$

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + \boldsymbol{W}\boldsymbol{x}^*\theta + \boldsymbol{K}\boldsymbol{x}^*0 + (\boldsymbol{\varepsilon} - \boldsymbol{U}\boldsymbol{x}^*\theta - \boldsymbol{C}\boldsymbol{x}^*\theta)$$

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + \boldsymbol{W}\boldsymbol{x}^*\theta + \boldsymbol{K}\boldsymbol{x}^*0 + (\boldsymbol{\varepsilon} - \boldsymbol{U}\boldsymbol{x}^*\theta - \boldsymbol{K}\boldsymbol{x}^*c\theta)$$

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + \boldsymbol{W}\boldsymbol{x}^*\theta + \boldsymbol{K}\boldsymbol{x}^*(-c\theta) + (\boldsymbol{\varepsilon} - \boldsymbol{U}\boldsymbol{x}^*\theta)$$

This is a substantially changed result. As before, there will still be attenuation bias on θ that results from the measurement error in **W**. Now, however, the additional bias resulting from the expectation of **u*** being non-zero is transferred to the coefficient of **Kx*** rather than staying in the error term and affecting inferences in the rest of the model. Thus, scholars concerned about the misspecification of **W** should estimate a full model:

$$\boldsymbol{y} = \alpha + \boldsymbol{x}^*\beta + \boldsymbol{W}\boldsymbol{x}^*\theta_1 + \boldsymbol{K}\boldsymbol{x}^*\theta_2 + \boldsymbol{\varepsilon}$$

Kx*, hereafter referred to as the **Kx** or the K test, can serve as a specification test of **W** within the regression model. If one assumes that **Kx** - or its counterparts **Ky** and $\mathbf{K}\boldsymbol{\varepsilon}$ in other spatial regression models - is not a part of the data-generating process, then the coefficient of the K test should be zero in expectation and statistically significant only by chance.[1] If,

[1]This should not be a controversial assumption, as scholars rarely, if ever, estimate a spatial regression model that hypothesizes **Kx**, **Ky**, or $\mathbf{K}\boldsymbol{\varepsilon}$ as part of the data-generating process. This scholar views these models with severe skepticism, as it assumes that all observations are connected in a cross-section, regardless of their distance from other units. A model more true to the data-generating process could be specified by incorporating some measure of either distance or contiguity in a more nuanced manner. This new specification of **W**, of course, could then be tested for measurement error as previously described.

however, the elements of $\mathbf{W}$ have been misspecified by a constant in expectation, then the coefficient of the K test should be non-zero and statistically significant at rates greater than expected by chance. Thus, statistical significance of the coefficient of the K test should be viewed as evidence that $\mathbf{W}$ is misspecified and that the scholar has failed to adequately model spatial relationships within the data. Furthermore, the direction of the misspecification can be derived using both θ_2 and θ_2 by following the rules provided in Table 1.

The K test can validate both absolute and relative relevancy determinations within $\mathbf{W}$. The test is designed to validate relative relevancy determinations within $\mathbf{W}$, as the value of its coefficient is derived by the change in expectation for the non-diagonal elements within $\mathbf{W}$ due to misspecification. But it can also pick up on absolute relevancy determinations as well. After all, incorrectly determining a particular w_{ij} to be zero when it is truly non-zero will change the expectation of the non-diagonal elements of $\mathbf{W}$. Except in the knife-edge case where incorrect absolute relevancy determinations cancel out and do not lead to a change in the expectation of the non-diagonal elements of $\mathbf{W}$, the K test also validates absolute relevancy determinations. This important feature makes it a clear improvement over current tests.

This K test can be seen as a generalization of the one described earlier by Neumayer and Plumper (2016). In their test, $\boldsymbol{W^2}$ contains a one in w_{ij2} observation when w_{ij1} equals zero, and zero otherwise. My test includes a one for all non-diagonal elements, regardless of whether its counterpart assigns a particular w_{ij} to be zero. This allows for a test of both the absolute relevance and relative relevance determinations within $\mathbf{W}$. An argument could be made for extending their approach and separating the tests for absolute relevance and relative relevance. But this would only be justifiable if one expected that the degree of accuracy between the absolute relevance specifications and relative relevance specifications in $\mathbf{W}$ are different. Otherwise, estimating them in a single parameter would be more efficient.

As a regression-based test, the results of the K-test are conditional upon an otherwise well-specified regression model. A poorly specified regression model may bias the inferences drawn from the test. For example, it is entirely possible that multiple spatial processes generated the data and that the scholar only modelled all of them. In this instance, the test could very well pick up on the omitted spatial process even if the **W** for the modelled spatial relationship is correctly specified. It is unclear how many data-generating processes in the social sciences can be characterized by multiple spatial processes. Similarly, it is unclear whether a scholar is more likely to omit a spatial process rather than misspecify **W**. Regardless, statistical significance of the K-test indicates a researcher has not adequately captured the spatial relationships within the data. In this way, the test is similar to Ramsey's RESET test (1969).

Yet the researcher does not have to stop there when the specter of multiple spatial processes looms large. Scholars can test the null hypothesis that the coefficient of the spatial lag, θ_1, is same when estimating the regression model both with and without the K-test (Clogg, Petkova, and Haritou 1995). If the null is rejected, the scholar has reason to believe that either **W** had measurement error or a spatial process correlated with **W** has been omitted. Note that this process is not useful for a spatial lag of **y**; an omitted spatial lag of **y** will automatically be correlated with the other independent variables in the model, even other spatial lags of **y**.

2.3 Limits and Considerations in Cross-Sectional and Panel Data Settings

While the K-test is theoretically appealing, it suffers from important limitations to its use in cross-sectional settings. In the SLX model, the sum of **Kx** and **x** results in a constant. In order to implement this test in the SLX context, then, a researcher has two options. The researcher can omit **x**, which results in omitted variable bias in almost all applied settings and, if $\mathbf{x}$ has spatial autocorrelation similar to $\mathbf{y}$, introduces false positives in the test. The researcher can also omit the constant, resulting in smearing and also resulting in false positives. In both scenarios, the test is no longer theoretically defensible without making overly restrictive assumptions. The problem, however, is not limited to the SLX case. The use of this test in the SAR case when also estimating a constant results in inconsistent estimates due to the limited variation within the data (Kelejian and Prucha 2002, Lee 2004). [2]

This limitation, however, is mitigated when extending beyond a single cross-section, such as in panel data. If $\mathbf{x}$ varies over both time and space and $\mathbf{Wx}$ is a purely spatial lag[3], then the sum of $\mathbf{x}$ and $\mathbf{Kx}$ does not result in a constant. This allows for the estimation of $\mathbf{x}$, $\mathbf{Kx}$, and the constant in the same regression model. The same is true in the SAR setting (Kelejian and Prucha 2002, Kelejian, Prucha, and Yuzefovich 2006).Thus while the test of the correct of $\mathbf{W}$ can only be theoretically discussed in the cross-sectional context, the test can be used by applied researchers in the context of multiple cross-sections.

But the use of the test in these applications also has unique considerations associated with it. When a spatial lag is taken of $\mathbf{x}$, where x varies over both time and space, then the sum of $\mathbf{x}$ and $\mathbf{Kx}$ results in a new variable, $\mathbf{x}_{total}$, that only varies over time. This makes the test potentially sensitive to temporal unit effects, both that are uncorrelated and correlated with $\mathbf{x}_{total}$. I offer suggestions for each case in turn. If there are temporal unit effects that are uncorrelated with $\mathbf{x}_{total}$ but are not accounted for in the model, $\mathbf{Kx}$ will likely result in Type 1 error and reject the $\mathbf{W}$ specification even if it is true. Fortunately, however, it is relatively easy to model such unit effects using the random effects model.

[2] Unfortunately, this limitation also applies to Neumayer and Plumper's test. Their test suffers from the same multicollinearity problems just described in the binary contiguity setting because it is a restricted version of the more general test in this paper. In a setting in which $\mathbf{W}$ contains both absolute relevance and relative relevance determinations, such as an inverse distance specification with a cut-off distance, however, the test has power even when a constant is estimated. Thus the test can be used in these circumstances.

[3] As opposed to a lag over time and space.

More problematic is the possibility of temporal unit effects that are correlated with $\mathbf{x}_{total}$. In a normal setting with multiple cross-sections, a scholar could combat this problem using fixed effects. The lack of spatial variation in $\mathbf{x}_{total}$, however, prevents the inclusion of temporal fixed effects within SLX models. The inclusion of fixed effects when using $\mathbf{Ky}$ has similar problems, as the estimates are both biased and inconsistent (Kelejian, Prucha, and Yuzefovich 2006). As it stands, there is no readily available solution to including both my specification test and temporal fixed effects. Scholars must assume an otherwise well-specified regression model, at least on the temporal dimension, in order to make inferences about the spatial relationships within the data.

3 Simulation

In order to test my claims, I employ two sets of Monte Carlo simulations: one featuring cross-sectional data and the other featuring panel data. One thousand data points are generated that either follows a simple SAR data-generating process or a SLX data generating process:

$$\boldsymbol{y} = \boldsymbol{x}\beta + \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{y} = \boldsymbol{x}\beta + \theta \boldsymbol{W}\boldsymbol{x} + \boldsymbol{\varepsilon}$$

where $\beta = 5$ and $\rho = \theta = 0.75$. $\boldsymbol{\varepsilon}$ is drawn from a standard normal distribution, while $\boldsymbol{x}$ is generated from a uniform distribution between -1 and 1. As per the above discussion, the constant is omitted in the cross-sectional data-generating process in order to test its use. In panel data setting, a constant α is included in each data-generating process, with $\alpha = 10$.[4] A variety of $\mathbf{W}$ specifications are used in the data generating processes, as discussed below.

I then proceed to test my K test of the specification of $\mathbf{W}$ in a variety of data-generating

[4] In the panel data-generating processes, n=100 and t=10, resulting in 1000 observaions to match the number in the cross-sectional context.

processes for the SAR and SLX models given above. A spatial regression model is estimated on the data that also employs my test of the specification of **W**, which includes either **Ky** or **Kx** depending on the regression model.[5] The **W** matrix used in estimating the models is either the correct or an incorrect specification, with an incorrect specification depending upon the particular data-generating process employed. This process is repeated one thousand times, and the average bias for the coefficient estimates and their coverage probabilities are subsequently reported. As implied by the data-generating processes, the true coefficient value for either **KX** or **Ky** is zero. In order for my test to be valid, bias of the estimated coefficients of **KX** and **Ky**, ρ_2 or θ_2, when **W** is correctly specified should, on average, be zero and the coverage probability should be the size of the hypothesis test employed, which in this paper is 95 percent. Likewise, bias in the coefficient estimates when **W** is misspecified should be nonzero and the coverage probability should be less than 95 percent, with both the magnitude of the bias increasing and the coverage probability decreasing as the level of misspecification increases.

A number of specifications of **W** are used in different data generating processes and, subsequently in order to reflect the variety of situations that scholars confront. Subsequently, a wide variety of mispecifications of **W** are tested, depending on the particular data generating process. These are both described in Table 2. To begin, I randomly generate a **W** matrix and then see how the test fairs using the correct **W** matrix and other randomly generated **W** matrices. I then test matrices that are misspecifications because they are the correct **W** plus a constant. I finally test a binary contiguity specification, using the sphere of influence specification strategy, in which the size of the sphere relative to the true specification is modified by a constant. Code for the simulation will be released upon publication.

[5]OLS is used to estimate the SLX models, while 2SLS is used to estimate the SAR models.

3.1 The K test for the Specification of W

Table 3 presents the basic simulation results for both a correctly specified and a randomly specified **W** in a SLX and SAR data-generating process using either cross-sectional or panel data, respectively. When **W** is correctly specified, the K test's coefficient centers around zero and is rarely significant. When the specification of W is random, however, there is an appreciable bias to zero and the test's coefficient is statistically significant. In most of the models with a random W, the coverage probability is just above zero. The sole exception is spatial autoregressive models on a single cross-section. In this model, the test is statistically different from zero in about half of the models. This is considerably less power than in other circumstances. But with the appreciable size of the test, the interpretation of the test's coefficient still applies: when the test's coefficient is statistically distinguishable from zero, then there is strong evidence that W is misspecified.

Tables 4 and 5 present the results for misspecifying a **W** matrix by a constant for cross-sectional and panel data generating processes, respectively. The results reflect those reported above. For SLX data-generating processes and a SAR data-generating process using panel data, the test's coefficient is rarely statistically distinguishable from zero when W is correctly specified and almost always statistically distinguishable from zero when W is incorrectly specified. Again, however, the test is much less powerful for a SAR data-generating process using cross-sectional data. The Tables also confirm the guidance in Table 1. When both ρ and the test's coefficient are positive, the elements of **W** have been deflated. When ρ is positive and the test's coefficient is negative, though, the elements of **W** have been inflated.

Tables 6 and 7 present the results of data-generating processes characterized by a sphere of influence construction for cross-sectional and panel data, respectively, where the radius of the sphere may be misspecified by a constant. Here the simulations are more diverse in their results. The test's discrimination abilities shown in Table 6 are largely the same as in previous results: the test is considerably powerful in the SLX case but much less so in the SAR case. The test's coefficient, however, is biased in the same direction whether the radius of the sphere is too small or too large, inconsistent with the previous results.

In Table 7, the test is shown to be largely powerful when the size of the sphere is too small. The test loses power, however, when the size of the sphere is too large; in the SLX case in particular, the test has extremely little power. This is unusual, as up to this point the test has always worked well in the SLX case regardless of the number of cross-sections under review. Additional Monte Carlo analysis is necessary to determine whether this is a property of the test in binary contiguity settings or whether it is a fluke. But like the results in Tables 4 and 5, Table 7 confirms the guidance given by Table 1 as to the nature of the misspecification of **W**.

4 Empirical Application: Williams and Whitten (2015)

In order to demonstrate the test in use, I apply it to a study published by Williams and Whitten (2015). The authors argue that changes in a party's electoral fortunes will be dependent upon the fortunes of other parties, with correlation becoming stronger as the ideological similarity between parties increases. To test this theory, they estimate a model of a party's change in electoral vote share from election to election in 23 countries from 1951-2005. This is modelled as a function of covariates including a spatial lag of the dependent variable. According to their theory, the spatial dependence between parties in low-clarity elections, where party responsibility for the state of the economy is unclear, will be stronger than parties in high-clarity elections, where party responsibility for the state of the economy is clear. After estimating two spatial regression models, the authors indeed find that while both sets of observations exhibit spatial dependence, parties in low-clarity elections exhibit greater spatial dependence than parties in high-clarity ones.

Whitten and Williams generally construct **W** in their models to have non-zero values along the block diagonal, or that parties will only be spatially related that are in the same country and in the same time period. They use data from the Comparative Manifestos Project to construct distance between parties on a single dimension. Four different specifications of **W** are considered according to a 2x2 specification. The authors vary in using either absolute distance or squared distance between parties. The authors also vary in allowing all parties in a given year and time period to be non-zero or only contiguous parties, or the parties ideologically adjacent to a particular party. Using information criteria, the authors find that the **W** constructed using absolute distance and only contiguous best fits the data and use this matrix in their analysis.

While the information criteria do identify an ideal **W** specification out of the set of choices, it is not clear whether the final specification used is valid. Because Williams and Whitten are using panel data, however, we can apply the K test to the spatial regression models for both low-clarity and high-clarity elections to evaluate if there is a specification issue and, if so, correct for it in the analysis. Table 8 contains both a replication of the authors models, as well as an estimation of those models using the K-test just described.[6]

In the model of high-clarity elections, the K-test does not indicate measurement error in the specification of **W**. The test, though having a larger coefficient than spatial lag of Williams and Whitten, is not statistically significant at any conventional level. The spatial lag of Williams and Whitten also does not appear to change much between the two models. I would conclude from this evidence that the specification of **W** is valid given the evidence

[6]Readers familiar with Williams and Whitten (2015) will note that Table 8 does not contain a number for number replication of their results. This is due to the spatial regression software used, a user-written STATA package using MLE by Pisati (2001). The package could not allow for the estimation of multiple spatial lags, so different software programs were used. However, both the STATA 15 spatial regression suite and a user-written program by Hays, Kachi, and Franzese (2010), both of which use MLE, provide different results from the Pisati package but identical results to each other. The results indicate that, if error exists, it is likely in the Pisati package. Since the substantive inferences with either set of the results is the same, however, there is little reason to worry about this possibility. The program by Hays, Kachi, and Franzese (2010) is used for the present analysis.

in the data.

In the model of low-clarity elections, however, there is evidence that the spatial lag offered by Williams and Whitten does not capture all of the spatial relationships within the data. The K-test's coefficient is statistically significant at virtually all conventional standards and, as expected, there is a significant difference between the spatial lag in the original model and the spatial lag when the K-test is estimated. As both the coefficient of the original spatial lag and the K-test are negative, this indicates that, in the presence of measurement error, a negative constant was added to the spatial relationships in $\mathbf{W}$.

The directional inferences made by Williams and Whitten (2015) are robust to the error detected by the K-test. Low-clarity elections exhibit more spatial dependence than high-clarity elections, even in the presence of K-test in both models. Additionally, as the replication materials show, the confidence intervals of the two estimates do not overlap, which the authors use as heuristic to argue that the coefficients are not different by chance.

But the magnitude of the difference between models shrank in the presence of the K-test, a notable results given scholars increasing attention to effect sizes. In other contexts, the change in coefficients may be large enough to affect directional inferences, with coefficients either losing statistical significance or changing signs.

5 Discussion

This paper contributes to the literature on spatial regression models by arguing that the uncertainty surrounding the specification of $\mathbf{W}$ is inherently a measurement error problem. This argument leads to two important contributions. First, it clearly states the bias that results when $\mathbf{W}$ is misspecified in regression models. Not only does the misspecification of $\mathbf{W}$ lead to attenuation bias, but it also leads to other forms of bias that are unique to spatial regression models. While this unique bias is difficult to predict ex ante, it becomes easier to discuss when scholars take this measurement error into account and try to model it.

Second, it reveals a theoretically appealing test of the specification of **W**, which I call the K test. It jointly examines the validity of the absolute relevance and relative relevance determinations within **W**. While perfect multicollinearity and other estimation issues prevents the tests' use in applied settings using cross-sectional data, it can easily be implemented when using data with multiple cross-sections, such as panel data. The Monte Carlo experiments validate that the test is useful in a variety of settings, while the application to Williams and Whitten (2015) show how it might be applied in practice.

Scholars must exercise some care when using the K test, specifically when adjudicating between multiple **W** matrices. If the test is used multiple times, the results must be weighted by the number of spatial lags estimated in order to avoid spurious inferences that occasionally result when using significance tests. Alternatively, this problem can be avoided by using the test in conjunction with other methods of adjudicating between multiple **W** matrices, like the J test. This would result in a two-step procedure: identify the best choice of **W** among a set of alternatives and then test to see if that specification is valid.

The frame of measurement error should inform future research into dependencies among units in regression models. Investigation should be given to the application of test developed here beyond modelling spatial dependencies in data. For example, time series models can also be expressed using **W** matrices. It would be reasonable to assume that the test could also be applied in these settings, and may even be more widely used given unidirectional nature of time. Indeed, many tools for diagnosing and estimating spatial dependencies may be useful in the temporal context. Research on these possibilities, however, is still in the conceptual stages.

References

[1] Bell, Andrew, and Kelvyn Jones. 2015. "Explaining fixed effects: Random effects modeling of time-series cross-sectional and panel data." *Political Science Research and Methods* 3(1): 133-153.

[2] Brambor, Thomas, William R. Clark, and Matt Golder. 2006. "Understanding interaction models: Improving empirical analyses." *Political Analysis* 14(1): 63-82.

[3] Case, Anne. 1992. "Neighborhood influence and technological change." *Regional Science and Urban Economics* 22(3): 491-508.

[4] Clogg, Clifford C., Eva Petkova, and Adamantios Haritou. (1995). "Statistical methods for comparing regression coefficients between models." *American Journal of Sociology* 100(5): 1261-1293.

[5] Corrado, Luisa and Bernard Fingleton. 2012. "Where is the economics in spatial econometrics?". *Journal of Regional Science* 52(2): 210-239.

[6] Hays, Jude C., Aya Kachi, and Robert Franzese. 2010. "A spatial model incorporating dynamic, endogenous network interdependence: A political science application." *Statistical Methodology* 7(3): 406-428.

[7] Harris, Richard, John Moffat, and Victoria Kravtsova. 2011. "In search of 'W'." *Spatial Economic Analysis* 6(3): 249-270.

[8] Kelejian, Harry H. and Ingmar R. Prucha. 2002. "2SLS and OLS in a spatial autoregressive model with equal spatial weights." *Regional Science and Urban Economics* 32(6): 691-707.

[9] Kelejian, Harry H., Ingmar R. Prucha, and Yevgeny Yuzefovich. 2006. "Estimation problems in models with spatial weighting matrices which have blocks of equal elements." *Journal of Regional Science* 46(3): 507-515.

[10] Lee, Lung-Fei. 2004. "Asymptotic Distributions of Quasi-Maximum Likelihood Estimators for Spatial Autoregressive Models." *Econometrica* 72(6): 1899-1925.

[11] Leenders, Roger T. A. 2002. "Modeling social influence through network autocorrelation: constructing the weight matrix." *Social networks* 24(1): 21-47.

[12] LeSage, James P., and Manfred M. Fischer. 2008. "Spatial growth regressions: model specification, estimation and interpretation." *Spatial Economic Analysis* 3(3): 275-304.

[13] LeSage, James P., and R. Kelley Pace. 2014. "The biggest myth in spatial econometrics." *Econometrics* 2(4): 217-249.

[14] McMillen, Daniel P. 2012. "Perspectives on spatial econometrics: linear smoothing with structured models." *Journal of Regional Science* 52(2): 192-209.

[15] Mundlak, Yair. 1978. "Pooling of Time-series and Cross-section Data." *Econometrica* 46(1): 69-85.

[16] Neumayer, Eric and Thomas Plumper. 2012. "Conditional Spatial Policy Dependence: Theory and Model Specification." *Comparative Political Studies* 45(7): 819-849.

[17] Neumayer, Eric and Thomas Plumper. 2016. "W." *Political Science Research and Methods* 4(1):175-193.

[18] Parent, Olivier, and James P. LeSage. 2008. "Using the variance structure of the conditional autoregressive spatial specification to model knowledge spillovers." *Journal of Applied Econometrics* 23(2): 235-256.

[19] Pisati, Mauirizio. 2001. “Tools for spatial data analysis.” *Stata Technical Bulletin* 60 (March): 21-36.

[20] Ramsey, J.R. 1969. “Tests for Specification Errors in Classical Linear Least Squares Regression Analysis.” *Journal of the Royal Statistical Society, Series B* 31(2): 350–371.

[21] Stakhovych, Stanislav and Tammo Bijmolt. 2009. “Specification of spatial models: A simulation study on weights matrices.” *Papers in Regional Science* 88(2): 389-408.

[22] Tobler, W. R. 1970. “A Computer Movie Simulating Urban Growth in the Detroit Region.” *Economic Geography* 46: pp. 234-240.

[23] Halleck Vega, Solmaria, and J. Paul Elhorst. 2015. “The SLX model.” *Journal of Regional Science* 55(3): 339-363.

Table 1: The Direction of the Measurement Error in **W**, c, by the values of θ_1 and θ_2

	$\theta_2 > 0$	$\theta_2 < 0$
$\theta_1 > 0$	$c < 0$	$c > 0$
$\theta_1 < 0$	$c > 0$	$c < 0$

Table 2: List of **W** matrices examined and misspecifications applied to them.

W name	Description	Description of Misspecification
Random	Observations are randomly located within space, and the **W** matrix contains the linear distance between observations.	Another **W** matrix randomly generated under the same conditions is used instead.
Constant Inflated	Spatial relationships between observations are randomly generated.	A constant is added to the **W** matrix
Sphere of influence	Observations are randomly located within space. A distance m is then defined and used to define neighbors: if an observation is no more than m distance away from another observation, then and only then are the two observations neighbors. Neighbors are coded as 1 in **W**, while non-neighbors receive 0.	The distance m is inflated by a constant.

Table 3: The K test and bias in SLX and SAR models using cross-sectional and panel data, respectively (Coverage Probabilities in parentheses)

		Cross-Section			Panel			
	W type	β	θ/ρ	K-Test	α	β	θ/ρ	K-Test
SLX	Correct	-0.00	0.00	-0.00	0.00	-0.00	0.00	-0.00
		(0.95)	(0.95)	(0.95)	(0.95)	(0.95)	(0.95)	(0.95)
	Random	0.00	-0.75	0.48	0.15	0.05	-0.75	0.47
		(1.00)	(0.00)	(0.00)	(0.09)	(0.93)	(0.00)	(0.00)
SAR	Correct	0.00	-0.00	-0.01	0.05	0.00	-0.00	-0.00
		(0.95)	(0.95)	(0.95)	(0.96)	(0.96)	(0.96)	(0.96)
	Random	0.00	-0.77	0.77	-0.21	0.08	-0.76	0.76
		(0.95)	(0.56)	(0.54)	(1.00)	(1.00)	(0.01)	(0.01)

Table 4: The K test and bias in SAR and SLX Models using cross-sectional data when **W** is misspecified by a constant (Coverage Probabilities in parentheses)

		-45	-25	0	+25	+45
SLX	β	-0.00	-0.00	-0.00	0.00	-0.00
		(0.94)	(0.94)	(0.96)	(0.94)	(0.95)
	θ	0.00	-0.00	0.00	-0.00	-0.00
		(0.94)	(0.95)	(0.95)	(0.95)	(0.95)
	K-Test	33.75	18.75	-0.00	-18.75	-33.75
		(0.00)	(0.00)	(0.94)	(0.00)	(0.00)
SAR	β	0.00	-0.00	-0.00	0.00	-0.00
		(0.95)	(0.94)	(0.96)	(0.95)	(0.95)
	ρ	-0.51	-0.27	0.05	0.38	0.67
		(0.38)	(0.89)	(0.95)	(0.95)	(0.93)
	K-Test	0.50	0.25	-0.08	-0.41	-0.70
		(0.34)	(0.88)	(0.94)	(0.95)	(0.93)

Table 5: The K test and bias in SAR and SLX Models using panel data when **W** is misspecified by a constant (Coverage Probabilities in parentheses)

		-45	-25	0	+25	+45
SLX	α	0.00	-0.00	0.00	-0.00	-0.00
		(0.94)	(0.95)	(0.95)	(0.96)	(0.96)
	β	-0.00	0.00	0.00	-0.00	0.00
		(0.94)	(0.95)	(0.96)	(0.94)	(0.95)
	θ	0.00	0.00	0.00	0.00	0.00
		(0.95)	(0.95)	(0.95)	(0.95)	(0.96)
	K-Test	33.75	18.75	-0.00	-18.75	-33.75
		(0.00)	(0.00)	(0.95)	(0.00)	(0.00)
SAR	α	0.35	0.22	0.26	0.19	0.29
		(0.96)	(0.95)	(0.96)	(0.95)	(0.95)
	β	0.00	-0.00	-0.00	-0.00	-0.00
		(0.95)	(0.95)	(0.96)	(0.95)	(0.96)
	ρ	-0.53	-0.30	0.00	0.31	0.55
		(0.00)	(0.00)	(0.95)	(0.00)	(0.00)
	K-Test	0.54	0.30	-0.01	-0.31	-0.56
		(0.00)	(0.00)	(0.96)	(0.00)	(0.00)

Table 6: The K test and bias in SAR and SLX Models using cross-sectional data when **W** uses a sphere of influence specification and is misspecified by a constant (Coverage Probabilities in parentheses)

		-30	-20	-10	0	10	20	30
SLX	β	0.50	0.54	0.48	0.00	0.39	0.52	0.43
		(0.07)	(0.02)	(0.12)	(0.95)	(0.56)	(0.06)	(0.43)
	θ	-0.78	-0.78	-0.59	-0.00	-0.50	-0.73	-0.53
		(0.00)	(0.00)	(0.00)	(0.95)	(0.00)	(0.00)	(0.00)
	K-Test	1.16	1.17	0.90	0.00	0.76	1.11	0.89
		(0.00)	(0.00)	(0.00)	(0.96)	(0.00)	(0.00)	(0.00)
SAR	β	0.00	-0.00	0.00	-0.01	0.06	0.00	0.00
		(0.95)	(0.95)	(0.95)	(0.95)	(0.95)	(0.94)	(0.96)
	ρ	-0.70	-0.70	-0.55	-0.08	-0.80	-0.99	-0.66
		(0.49)	(0.75)	(0.87)	(0.96)	(0.91)	(0.91)	(0.93)
	K-Test	0.79	0.74	0.67	-2.38	12.97	1.33	0.78
		(0.54)	(0.75)	(0.88)	(0.96)	(0.92)	(0.91)	(0.93)

Table 7: The K test and bias in SAR and SLX Models using panel data when **W** uses a sphere of influence specification and is misspecified by a constant (Coverage Probabilities in parentheses)

		-30	-20	-10	0	10	20	30
SLX	α	-0.02	0.01	0.01	-0.00	-0.03	-0.01	0.02
		(1.00)	(1.00)	(1.00)	(0.95)	(0.99)	(1.00)	(1.00)
	β	-0.10	-0.04	-0.07	-0.00	0.05	0.16	0.07
		(0.99)	(1.00)	(0.96)	(0.95)	(0.99)	(0.83)	(1.00)
	θ	-0.45	-0.35	-0.22	0.00	-0.06	-0.05	-0.01
		(0.00)	(0.00)	(0.00)	(0.95)	(0.23)	(0.94)	(1.00)
	K-Test	0.45	0.35	0.22	-0.00	-0.02	-0.07	-0.14
		(0.00)	(0.00)	(0.00)	(0.94)	(0.97)	(0.77)	(0.76)
SAR	α	0.31	0.58	0.55	0.29	0.16	0.08	0.20
		(1.00)	(1.00)	(1.00)	(0.97)	(1.00)	(1.00)	(1.00)
	β	-0.03	0.03	0.03	-0.00	-0.08	-0.01	0.05
		(1.00)	(1.00)	(1.00)	(0.95)	(0.97)	(1.00)	(1.00)
	ρ	-0.57	-0.46	-0.30	0.00	0.24	0.52	0.83
		(0.00)	(0.00)	(0.00)	(0.97)	(0.01)	(0.03)	(1.00)
	K-Test	0.57	0.45	0.29	-0.01	-0.26	-0.53	-0.85
		(0.04)	(0.06)	(0.14)	(0.96)	(0.22)	(0.22)	(1.00)

Table 8: Replication of Williams and Whitten (2015), with and without the K-test (Standard Errors in Parentheses)

	High Clarity		Low Clarity	
ρ	-0.002	-0.002	-0.006	-0.004
	(0.001)	(0.001)	(0.001)	(0.001)
K-test		-0.010		-0.059
		(0.021)		(0.014)
Real GDP per capita growth	-0.258	-0.257	-0.023	-0.020
	(0.098)	(0.098)	(0.058)	(0.058)
Coalition Party x Growth	0.461	0.461	-0.049	-0.058
	(0.166)	(0.166)	(0.106)	(0.104)
PM's Party x Growth	0.546	0.541	0.273	0.264
	(0.190)	(0.190)	(0.109)	(0.108)
Party Shift$_t$	0.005	0.005	0.014	0.014
	(0.013)	(0.013)	(0.009)	(0.009)
Party Shift$_{t-1}$	0.012	0.012	0.031	0.030
	(0.014)	(0.014)	(0.009)	(0.009)
Time Left in CIEP	-0.030	-0.029	-0.013	-0.012
	(0.017)	(0.017)	(0.006)	(0.006)
Coalition Party x Time Left	0.055	0.055	0.037	0.035
	(0.026)	(0.026)	(0.013)	(0.012)
PM's Party x Time Left	-0.979	0.09	0.051	0.051
	(0.421)	(0.029)	(0.013)	(0.012)
Coalition Party	-4.097	-4.09	-1.650	-1.630
	(0.987)	(0.987)	(0.518)	(0.510)
PM Party	7.149	7.161	0.640	0.453
	(3.205)	(3.203)	(1.811)	(1.782)
Niche Party	-0.243	-0.250	0.428	0.349
	(1.189)	(1.189)	(0.544)	(0.536)
No. of Gov't Parties	0.158	0.154	0.158	0.182
	(0.210)	(0.211)	(0.151)	(0.149)
PM's Party x No. of Gov't Parties	-0.979	-0.976	-0.031	-0.011
	(0.421)	(0.421)	(0.358)	(0.352)
Vote$_{t-1}$	0.002	0.002	-0.033	-0.032
	(0.022)	(0.022)	(0.013)	(0.013)
PM's Party x Vote$_{t-1}$	-0.322	-0.321	-0.078	-0.073
	(0.066)	(0.066)	(0.037)	(0.036)
Niche Party x Vote$_{t-1}$	-0.125	-0.124	-0.077	-0.072
	(0.089)	(0.089)	(0.044)	(0.043)
Effective Number of Parties	-0.240	-0.230	-0.190	-0.197
	(0.287)	(0.288)	(0.126)	(0.124)
Majority Government			0.135	0.109
			(0.320)	(0.315)
PM's Party x Majority Government			-0.292	-0.310
			(0.733)	(0.721)
Constant	2.624	2.575	1.419	1.329
	(1.018)	(1.023)	(0.550)	(0.542)